PBW301: Traffic Engineering
Lecture 6: Traffic Assignment

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Traffic Network Modeling

- Physical Network Model, include:
  - Road Segments➔ Links
  - Intersections ➔ nodes
- Traffic Operations Models, include:
  - Traffic stream modeling
  - Shockwaves and queuing
  - Controls (such as signal controls)
- Traffic Assignment Models, include:
  - Vehicles Routing

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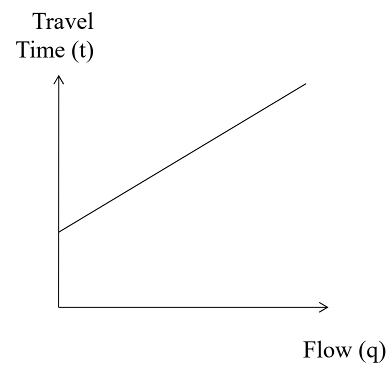
Traffic Network Modeling

- Traffic Assignment Modeling
  - Static Traffic Assignment
    - Ignores the time dimension
    - Assume that traffic characteristics are constant with time
  - Dynamic Traffic Assignment
    - Consider the time dimension
    - Consider the variability in traffic characteristics with time

Basic Concept

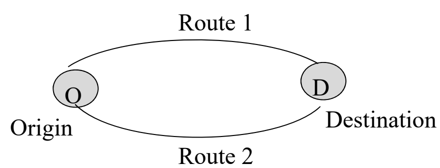
- Link Performance Function:
  - Travel time increases with the increase in traffic flow
  - In a linear case:
  - $t = a + b\,q$
  - At q=0, travel time equals the free flow travel time



User Equilibrium (UE) Traffic Assignment

- Basic Concept:
  - Travel Times on all routes, for a given OD pair, are equal for used routes and less than that of an unused route

If both routes are used:
➔Travel Time of Route 1
equals
Travel Time of Route 2



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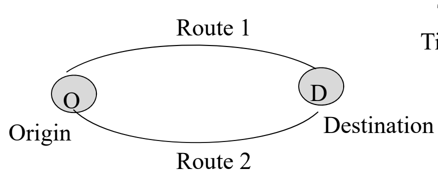
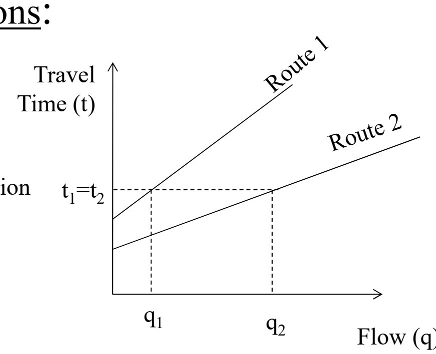
User Equilibrium (UE) Traffic Assignment

- Link Performance Functions:





If the total flow q_T then;
$q_T = q1 + q_2$
$t1 = t_2$

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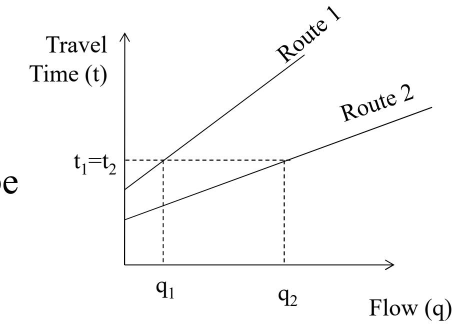
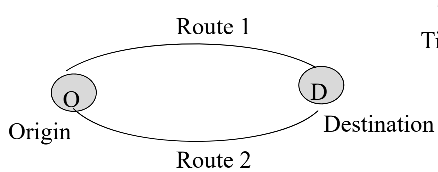
User Equilibrium (UE) Traffic Assignment

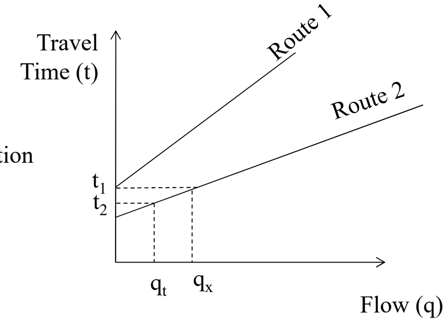
- Solving For UE:
  - Equate Routes Travel Time ($t_1=t_2$)
    - → $f(q_1)=f(q_2)$
  - Total flow (q_T) must be accommodated
    - → $q_T=q_1+q_2$



User Equilibrium (UE) Traffic Assignment

- Link Performance Functions:

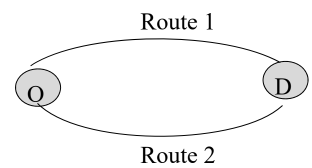




If the total flow is less than q_1 then;
All vehicles will use Route 2

User Equilibrium (UE) Traffic Assignment

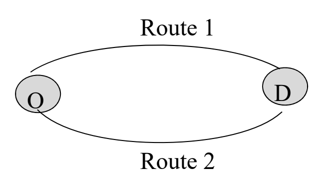
- Example 1 (UE):
  - Total Flow from A to B equals 10000 veh/hr
  - $t_1 = 30+0.0025x_1$
  - $t_2=15+0.0020x_2$
  - x is the link flow



User Equilibrium (UE) Traffic Assignment

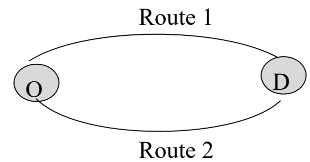
- Example 1 (UE):

$$q_T = q_1 + q_2 = 10000$$
$$t_1 = t_2$$
$$30 + 0.0025\, q_1 = 15 + 0.0020\, q_2$$
$$30 + 0.0025\, q_1 = 15 + 0.0020\,(10000 - q_1)$$
$$q_1 = 1111\, veh / hr$$
$$q_2 = 10000 - 1111 = 8889\, veh / he$$
$$t_1 = 30 + 0.0025\,(1111) = 32.8 \min$$
$$t_2 = 15 + 0.002\,(8889) = 32.8 \min$$
$$Total - Travel - Time = 10000\,(32.8) = 328000\, veh.\min$$



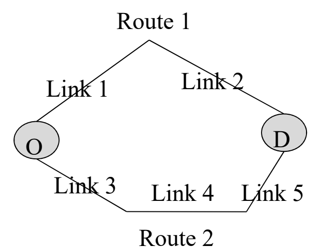
User Equilibrium (UE) Traffic Assignment

- Tips:
  - If a negative value is estimated for q_1, what does this mean:
    - No vehicles will use Route 1 (q_1=0)
    - All vehicles will use Route 2 (q_2=10000)



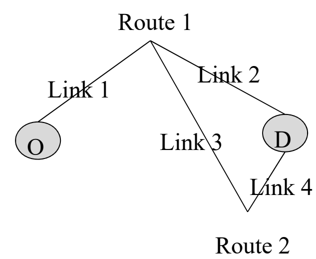
User Equilibrium (UE) Traffic Assignment

- Tips:
  - Travel Time of Route 1 = $t_{link1}+t_{link2}$
  - Travel Time of Route 2 = $t_{link3}+t_{link4}+t_{link5}$



User Equilibrium (UE) Traffic Assignment

- Tips:
  - Travel Time of Route 1 = $t_{link1}+t_{link2}$
  - Travel Time of Route 2 = $t_{link1}+t_{link3}+t_{link4}$
  - Traffic flow on link 1 equals the summation of traffic flows on route 1 and route 2



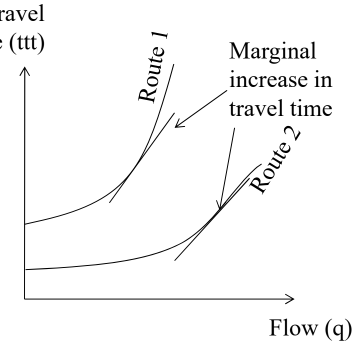
User Equilibrium (UE) Traffic Assignment

- Basic Concept:
  - Travel Times on all routes, for a given OD pair, are equal for used routes and less than that of an unused route.
- Assumptions:
  - Rational Drivers
  - Demand is Constant
  - Supply is Constant (constant capacity)
  - Drivers have perfect knowledge of routes and travel times

System Optimal (SO) Traffic Assignment

- Basic Concept:
  - Minimizing the Total Travel Time in the Network

For System Optimal Assignment, the marginal increase in travel times of all used routes (between a given OD pair) are equal and less than that of unused routes



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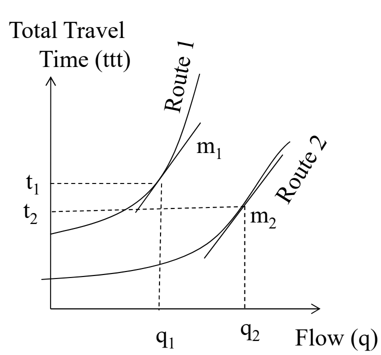
System Optimal (SO) Traffic Assignment

- Solving For SO:
  - Estimate Marginal function

$$m = \frac{d}{dq}(total - travel - time) = \frac{d}{dq}(q.t)$$

  - Equate estimated marginal increase of all routes ($m_1 = m_2$)
  - Total Flow must be accommodated ($q_T = q_1 + q_2$)



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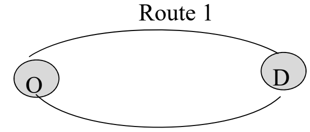
System Optimal (SO) Traffic Assignment

- Example 2 (SO):
  - Total Flow from A to B equals 10000 veh/hr
  - $t_1 = 30+0.0025x_1$
  - $t_2=15+0.0020x_2$
  - x is the link flow



System Optimal (SO) Traffic Assignment

- Example 2 (SO):

$$m_1 = \frac{d}{dq}(30x_1 + 0.0025x_1^2) = 30 + 0.005x_1$$

$$m_2 = \frac{d}{dq}(15x_2 + 0.002x_2^2) = 15 + 0.004x_2$$

$$q_T = q_1 + q_2 = 10000$$

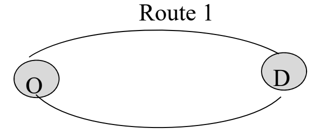
$$m_1 = m_2$$

$$30 + 0.005q_1 = 15 + 0.004q_2$$

$$30 + 0.005q_1 = 15 + 0.004(10000 - q_1)$$

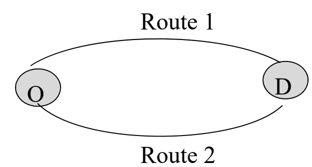
$$q_1 = 2778 veh/hr$$

$$q_2 = 10000 - 1111 = 7222 veh/hr$$



System Optimal (SO) Traffic Assignment

- Example 2 (SO):



$$q_1 = 2778 \, veh / hr$$
$$q_2 = 7222 \, veh / he$$
$$t_1 = 30 + 0.0025(2778) = 36.9 \min$$
$$t_2 = 15 + 0.002(7222) = 29.4 \min$$
$$Total - Travel - Time = 2778(36.9) + 7222(29.4) = 314835 \, veh.\min$$

Less than the UE case (328000 veh.min)

System Optimal (SO) Traffic Assignment

- Basic Concept:
  - Minimizing the Total Travel Time in the Network

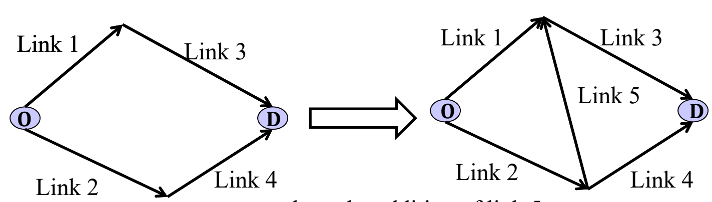
- Assumptions:
  - Drivers compliance
  - Demand is Constant
  - Supply is Constant (constant capacity)

UE & SO as Mathematical programs

- In large-scale traffic networks
  - Large number of ODs
  - Multiple alternative routes between each OD pair
- Mathematical Programming
  - Problem is formulated as a constrained minimization problem.
  - Just mathematical manipulation

Braess's Pseudo-Paradox

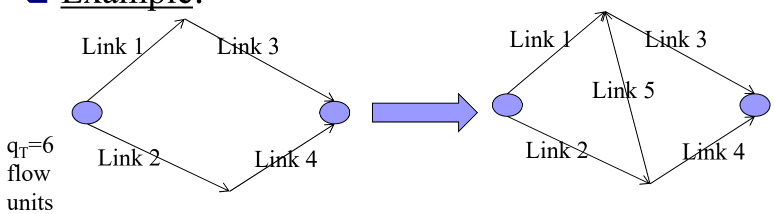
- Basic Concept:
  - An Increase in system capacity resulting in an increased network total travel time



case where the addition of link 5 caused an increase in total network travel time

Braess's Pseudo-Paradox

- Example:



$t_1 = 50 + x_1$
$t_2 = 50 + x_2$
$t_3 = 10x_3$
$t_4 = 10x_4$

$t_5 = 10 + x_5$

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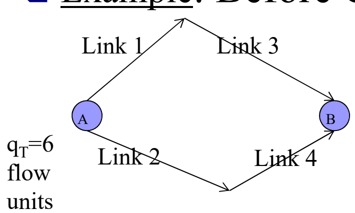
Braess's Pseudo-Paradox

- Example: Before Case UE



$t_1 = 50 + x_1$
$t_2 = 50 + x_2$
$t_3 = 10x_3$
$t_4 = 10x_4$

$t_{Route\ 1} = t_{Route2}$
$t_1 + t_3 = t_2 + t_4$
From Symmetry;
$q_1 = q_2 = 3$
$t_1 = t_2 = 53$
$t_3 = t_4 = 30$
$t_{route\ 1} = 83$
$t_{route\ 2} = 83$
Total Travel Time=6(83)=498

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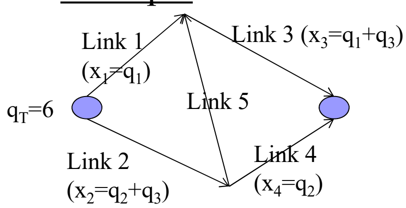
Braess's Pseudo-Paradox

- Example: After Case UE



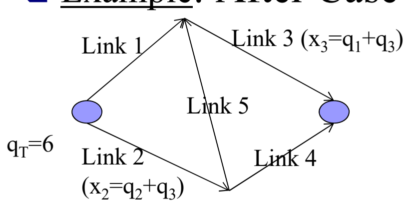
Available Routes:
Route 1 → links 1, and 3
Route 2 → links 2, and 4
Route 3 → links 2, 5, and 3

t_1= 50+x_1

t_2=50+x_2
t_3=10x_3
t_4=10x_4
t_5=10+x_5

Braess's Pseudo-Paradox

- Example: After Case UE

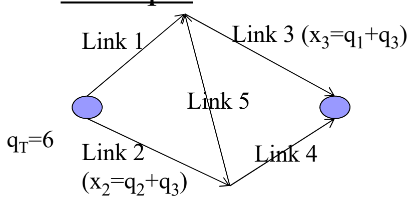


t_1= 50+x_1
t_2=50+x_2
t_3=10x_3
t_4=10x_4
t_5=10+x_5

$t_{Route\ 1}= t_{Route2}$ and $t_{Route\ 1}= t_{Route3}$
$t_1+t_3=t_2+t_4$ → eq.1
$t_1+t_3=t_2+t_5+t_3$ → eq.2
$q_T=q_1+q_2+q_3=6$ → eq.3
Solving 1, 2, and 3:
$q_1=q_2=q_3=2$
$t_{route\ 1}$=92
$t_{route\ 2}$=92
$t_{route\ 3}$=92
Total Travel Time=6(92)=552 More than the before case

Braess's Pseudo-Paradox

- <u>Example</u>: After Case SO



$t_1= 50+x_1$
$t_2=50+x_2$
$t_3=10x_3$
$t_4=10x_4$
$t_5=10+x_5$

<u>Another way to reach the same conclusion</u>:
→Solve for SO
→Results:
$q_1=3$
$q_2=3$
$q_3=0$

Braess's Pseudo-Paradox

- Not every addition in system capacity results in a reduction in network total travel time.

- Drivers choose routes to minimize their own travel time regardless system optimization.