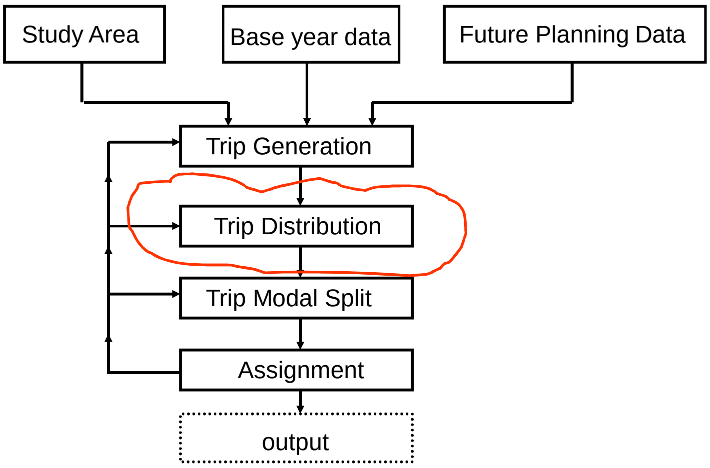


	i	j	k	l	
i	?	?	?	?	2200
j	?	?	?	?	1900
k	?	?	?	?	2000
l	?	?	?	?	2100
	1600	2400	2300	1900	

Productions

Attractions

- **Trip Production**

Household variables

Income

Family Structure

Car ownership

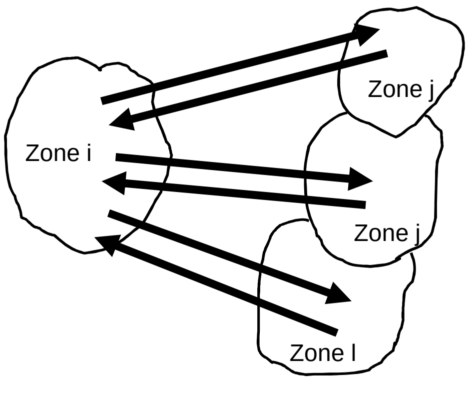
Accessibility

- **Trip Attraction**

Zone variables

Land use configuration

No. of jobs in the zone



	1	2	3	j	z	
1	T_{11}	T_{12}	T_{13}	T_{1j}	T_{1z}	O_1
2	T_{21}	T_{22}	T_{23}	T_{2j}	T_{2z}	O_2
3	T_{31}	T_{32}	T_{33}	T_{3j}	T_{3z}	O_3
i	T_{i1}	T_{i2}	T_{i3}	T_{ij}	T_{iz}	O_i
z	T_{z1}	T_{z2}	T_{z3}	T_{zj}	T_{zz}	O_z
	D_1	D_2	D_3	D_j	D_z	

$$O_i = \sum_j T_{ij}$$

$$D_j = \sum_i T_{ij}$$

$$T = \sum_{ij} T_{ij} = \sum_i O_i = \sum_j D_j$$

Matching Trip Productions and Attractions

	i	j	k	l	
i	?	?	?	?	2200
j	?	?	?	?	1900
k	?	?	?	?	2000
l	?	?	?	?	2100
	1600	2400	2000	1900	

7900

Productions does not match Attractions

Why:
Obtained from two different models

8200

$$\sum_i O_i \neq \sum_j D_j$$

In general, trip production models are better than their trip attraction counterparts

Why:
They use good household explanatory variables than trip attraction models which use zonal based variables

$$correction\ factor = \frac{\sum_i O_i}{\sum_j D_j} = \frac{8200}{7900}$$

Variables affecting Trip Distribution

- Travel Cost
- Out of pocket money (gas, tolls, transit fares)
- Intangible cost (car depreciation)

- Travel Time
- In-vehicle time
- Out-of-vehicle time (walking, waiting, transferring)

Generalized Cost

Cost-Based GC

$$GC_{ij} = C_{ij} + a * T_{ij}$$

$$GC_{ij} = C_{ij} + a^{in-veh} * T_{ij}^{in-veh} + a^{out-of-veh} * T_{ij}^{out-of-veh}$$

Have units of $/min

Value of in-vehicle time

Value of out-of-vehicle time

Time-Based GC

$$GC_{ij} = T_{ij} + b * C_{ij}$$

Min/$ - Duration of Money

Which one makes more sense?

- For the Same Destination

As income increases,

For cost-based GC

Value of time increases (daily_income/1440)

For time-based GC

Duration of money decreases (1440/daily_income)

- For the Same Destination

As income increases,
For cost-based GC
GC increases

For time-based GC
GC decreases

- For the Same Destination

As income increases,
For cost-based GC
Destination becomes less reachable

For time-based GC
Destination becomes more reachable

Growth Factor Methods

- Uniform Growth Factor
- Singly Constrained Growth Factor
- Doubly Constrained Growth Factor

Uniform Growth Factor

Given

(1) Current OD matrix

	1	2	3	4
1	t11	t12	t13	t14
2	t21	t22	t23	t24
3	t31	t32	t33	t34
4	t41	t42	t43	t44

(2) Current OD matrix

A uniform growth factor

Say "20%"

Due to:
-Population growth
-Economy growth

- **Advantages**

  Simple and easy to implement

- **Disadvantages**

  - hard to predict for expansion or addition of new zones in the study area.

  - assumes homogenous/uniform growth in the entire area of study.

  - needs information on the current OD matrix.

Singly Constrained Growth Factor

Given

(1) Current OD matrix

	1	2	3	4
1	t11	t12	t13	t14
2	t21	t22	t23	t24
3	t31	t32	t33	t34
4	t41	t42	t43	t44

(2) Future Trip Productions

O1
O2
O3
O4

Obtained from the trip generation step

We call it singly constrained because the following constraint has to be satisfied:

$$\sum_j t_{ij}^f = O_i^f \quad \forall i$$

Calculate a growth factor for each row

$$f_i = \frac{O_i^{future}}{\sum_j T_{ij}}$$

$$f_2 = \frac{O_2^{future}}{t_{21} + t_{22} + t_{23} + t_{24}}$$

- **Advantages**
- Simple and easy to implement.
- Relaxes the assumption of having homogenous/uniform growth in the entire area of study.

- **Disadvantages**

  - hard to predict for expansion or addition of new zones in the study area.

  - needs information on the current OD matrix.

Doubly Constrained Growth Factor

Given

(1) Current OD matrix

	1	2	3	4
1	t11	t12	t13	t14
2	t21	t22	t23	t24
3	t31	t32	t33	t34
4	t41	t42	t43	t44

(2) Future Trip Productions

O1
O2
O3
O4

(3) Future Trip Attractions

	D1	D2	D3	D4

We call it doubly constrained because the following two constraints have to be satisfied:

$$\sum_j t_{ij}^f = O_i^f \quad \forall i$$

$$\sum_i t_{ij}^f = D_j^f \quad \forall j$$

Furness' Algorithm (1965)

$$t_{ij}^f = gf_{ij} * t_{ij}^c$$

$$t_{ij}^f = a_i * b_j * t_{ij}^c$$

a_i = Growth factor due to productions
b_j = Growth factor due to attractions

Step 1

Assume b’s =1 and solve for a’s that satisfies the production constraints

Step 2

With the latest a’s, solve for b’s that satisfies the attraction constraints.

Step 3

keeping the b’s fixed, solve for a’s and repeat steps (2) and (3) until convergence.

Example

	1	2
1	200	700
2	300	100

1800
900

	1100	1600

Assume b's =1
200 * a1 * 1.0 + 700 * a1 * 1.0 = 1800
300 * a2 * 1.0 + 100 * a2 * 1.0 = 900
a1=2 and a2=2.25

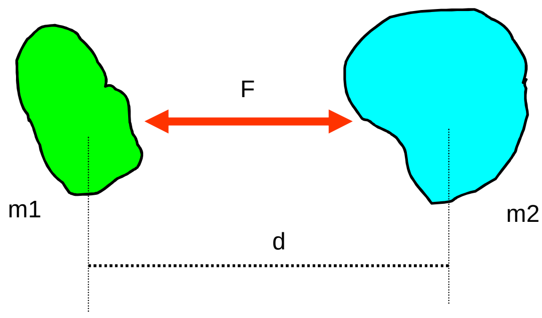
200 * b1 * 2 + 300 * b1 * 2.25 = 1100
700 * b2 * 2 + 100 * b2 * 2.25 = 1500
Solve two equations to calculate b1 and b2

200 * a1 * b1+ 700 * a1 * b2 = 1800
300 * a2 * b1+ 100 * a2 *b2 = 900
Solve two equations to calculate a1 and a2

	1	2
1	a_1b_1	a_1b_2
2	a_2b_1	a_2b_2

Continue until a's and b's are unchanged in two consequent iterations

The Gravity Distribution Model



Newton's Gravitational Law

$$F = C * \frac{m_1 * m_2}{d^2}$$

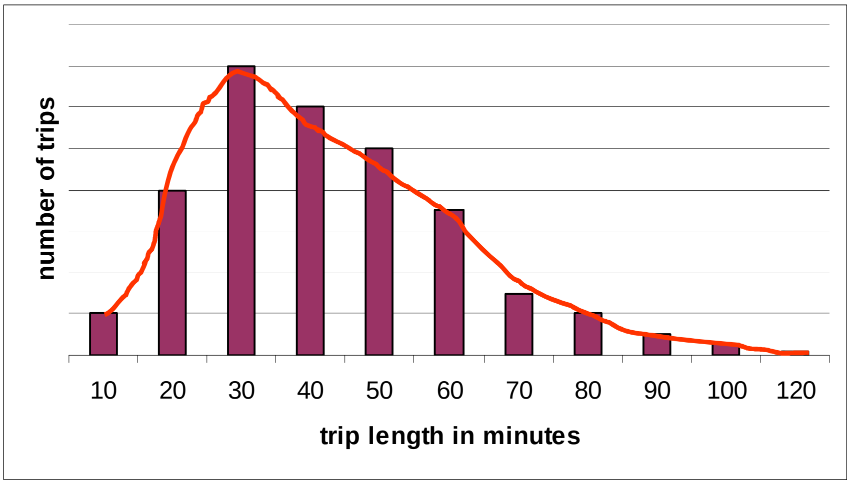
In analogy to Newton's Gravitational Model

$$T_{ij} = \alpha * \frac{O_i * D_j}{d_{ij}^2}$$

$$T_{ij} = \alpha * O_i * D_j * f(c_{ij})$$

Deterrence function

Trip Length Diagram



Practical Considerations

- Sparse Matrices

  Assume a study area with 500 zones (250000 cells).

  If the sampling rate is 20% (1 in 5), the chances of making no observations on a particular O-D pair are very high.

- Treatment of external zones

  In some cases, a significant portion of the trips start or end outside the study area.

  These trips could not be estimated using the gravity models.

  Usually, use the growth factor methods.