



## Space-time Diagram at signalized intersections

- Basic Concepts:
- Space-time Slopes: represent speeds
- Traffic State: an area with steady-state vehicle speeds (consistent space-time slopes)
- Shockwave speed ( $\omega$ ): is the slope of the boundary between two traffic states





## Shockwaves at signalized intersections

$\square$ Estimation of Queue Characteristics



## Shockwaves at signalized intersections

$\square$ Example: Signalized intersection

- State A:
$\mathrm{q}_{\mathrm{A}}=1000 \mathrm{veh} / \mathrm{hr}$
$q_{A}=1000=80 k_{A}-0.8 k_{A}^{2}$
$k_{A}=\frac{80 \pm \sqrt{(80)^{2}-4(0.8)(1000)}}{2(0.8)}$
$k_{A}=85.35 o r 14.64 \mathrm{veh} / \mathrm{km}$



## Shockwaves at signalized intersections

$\square$ Example: Signalized intersection

- State B:
- $\mathrm{q}_{\mathrm{B}}=0$
- $\mathrm{k}_{\mathrm{B}}=\mathrm{k}_{\mathrm{j}}=80 / 0.8=100 \mathrm{veh} / \mathrm{km}$
- State C:
- $\mathrm{q}_{\mathrm{C}}=\mathrm{q}_{\text {max }}=80 * 100 / 4=2000 \mathrm{veh} / \mathrm{hr}$
- $\mathrm{k}_{\mathrm{C}}=\mathrm{k}_{\mathrm{j}} / 2=100 / 2=50 \mathrm{veh} / \mathrm{km}$



## Shockwaves at signalized intersections

$\square$ Example: Signalized intersection
$\omega_{A B}=\frac{q_{A}-q_{B}}{k_{A}-k_{B}}=\frac{1000-0}{14.46-100}=-11.69 \mathrm{~km} / \mathrm{hr}$ $\omega_{B C}=\frac{q_{B}-q_{C}}{k_{B}-k_{C}}=\frac{0-2000}{100-50}=-40 \mathrm{~km} / \mathrm{hr}$



## Space-time Diagram along a highway with a slow moving truck

- Basic Concepts:
- State A: Free-Flow State
- State B: Vehicles_slowing down behind a slow truck
- State C: Vehicles flow with full section capacity



## Shockwaves along a highway with a slow moving truck

- Estimation of Shockwave Speed

$$
\left\lceil\begin{array}{l}
\omega_{A B}=\frac{q_{A}-q_{B}}{k_{A}-k_{B}} \\
\omega_{B C}=\frac{q_{B}-q_{C}}{k_{B}-k_{C}}
\end{array} \Longrightarrow \omega=\frac{\Lambda q}{\Delta k}\right.
$$

$\rightarrow+$ ve value for forward propagating shockwave -ve value for backward propagating shockwave


A to $B$ results in a Forming shockwave


## Shockwaves along a highway with a slow moving truck

$\square$ Example: slow moving truck

- Vehicles are flowing with a rate of $1000 \mathrm{veh} / \mathrm{hr}$
- A slow moving truck (with $20 \mathrm{~km} / \mathrm{hr}$ ) entered the highway and exists after 500 m .
- Estimate the speed of the forming/ clearing shockwaves.
- Estimated the max queue length.
- Estimated the distance the queue will reach
- Consider the model u=80-0.8k.


## Shockwaves along a highway with a slow moving truck

$\square$ Example: slow moving truck

- State A:
- $q_{A}=1000 \mathrm{veh} / \mathrm{hr}$
$q_{A}=1000=80 k_{A}-0.8 k_{A}{ }^{2}$
$k_{A}=\frac{80 \pm \sqrt{(80)^{2}-4(0.8)(1000)}}{2(0.8)}$
$k_{A}=85.35 \mathrm{or} 14.64 \mathrm{veh} / \mathrm{km}$



## Shockwaves along a highway with a slow moving truck

$\square$ Example: slow moving truck

- State B:
- $\mathrm{u}_{\mathrm{B}}=20 \mathrm{~km} / \mathrm{hr}$
- $\mathrm{k}_{\mathrm{B}}=(80-20) / 0.8=75 \mathrm{veh} / \mathrm{km}$
- $\mathrm{q}_{\mathrm{B}}=75(20)=1500 \mathrm{veh} / \mathrm{km}$
- State C:
- $\mathrm{q}_{\mathrm{C}}=\mathrm{q}_{\text {max }}=80 * 100 / 4=2000 \mathrm{veh} / \mathrm{hr}$

- $\mathrm{k}_{\mathrm{C}}=\mathrm{k}_{\mathrm{j}} / 2=100 / 2=50 \mathrm{veh} / \mathrm{km}$


## Shockwaves along a highway with a slow moving truck

$\square$ Example: slow moving truck

$$
\begin{aligned}
& \omega_{A B}=\frac{q_{A}-q_{B}}{k_{A}-k_{B}}=\frac{1000-1500}{14.64-75}=8.28 \mathrm{~km} / \mathrm{hr} \quad \text { Note the sign } \\
& \omega_{B C}=\frac{q_{B}-q_{C}}{k_{B}-k_{C}}=\frac{1500-2000}{75-50}=-20 \mathrm{~km} / \mathrm{hr}
\end{aligned}
$$

## Shockwaves along a highway with a slow moving truck

$\square$ Example: slow moving truck
$\omega_{A B}=8.28 \mathrm{~km} / \mathrm{hr}$
$\omega_{B C}=-20 \mathrm{~km} / \mathrm{hr}$

Time - Truck - on - road $=0.5 / 20=0.025 h r$
$Q_{\text {reach }}=0.5 \mathrm{~km}$
Мах - Quеие $=0.5-8.28 * 0.025=0.293 \mathrm{~km}$


