Traffic Engineering - Lecture 3: Traffic Stream Modeling

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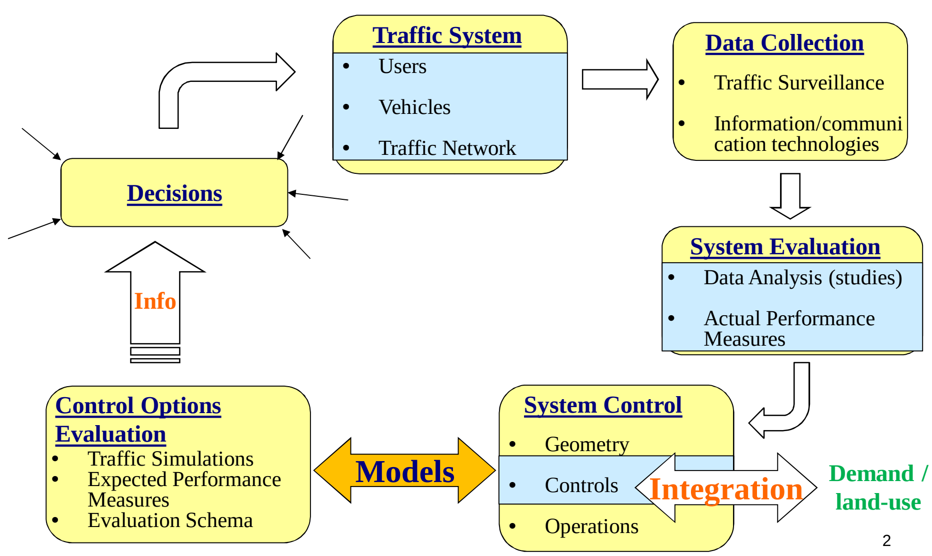
Course Syllabus

WHAT?
WHY?
HOW?



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Traffic Flow Modeling

- What?
  - Mathematical abstraction of traffic stream characteristics in uninterrupted flow situations
  - Uninterrupted flow: Traffic steams at roadway sections without operational controls
- Why?
  - Planning and Policy Analysis
  - Design Evaluation
  - Operations Management
  - Real-time decision support system

Traffic Flow Modeling

- How? Traffic Flow Modeling approaches
  - Macroscopic Approach: considers the traffic stream at a given section as one unit (similar to the flow of fluids).
  - Microscopic Approach: consider spacing and speeds of individual vehicles. Modeling individual vehicles behavior ultimately results in an aggregate traffic stream behavior.

Macroscopic Traffic Flow Models

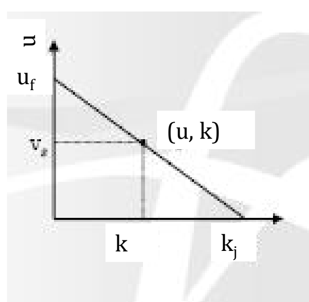
- Key Macroscopic Variables & Relationships
  - Flow (q): number of vehicles per time unit (veh/hr)
  - Speed (u): average speed (km/hr)
  - Density (k): number of vehicles per unit length (veh/km)
  - Key relationship:

$$q = k \cdot u \quad \rightarrow (1)$$

Greenshield Traffic Flow Model

- Fundamental Diagrams

$$u = u_f - \left(\frac{u_f}{k_j}\right)k \quad \rightarrow (2)$$



  - u_f: free flow speed
  - k_j: jam density

Greenshield Traffic Flow Model

- Fundamental Diagrams

$$u = u_f - \left(\frac{u_f}{k_j}\right)k \longrightarrow (2)$$

  - Multiply Both sides by u, substitute by q as in eq 1, and re-arrange

$$q = \left(\frac{k_j}{u_f}\right)(uu_f - u^2) \longrightarrow (3)$$

  - Multiply both sides by k, substitute by q as in 1, and re-arrange

$$q = u_f k - \left(\frac{u_f}{k_j}\right)k^2 \longrightarrow (4)$$

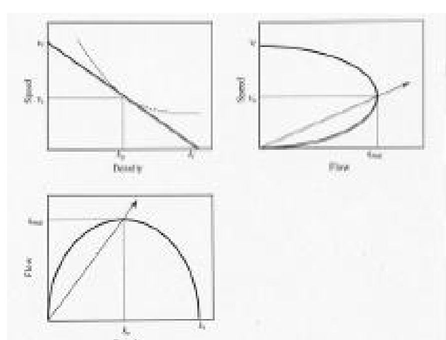
Greenshield Traffic Flow Model

- Fundamental Diagrams

$$u = u_f - \left(\frac{u_f}{k_j}\right)k$$

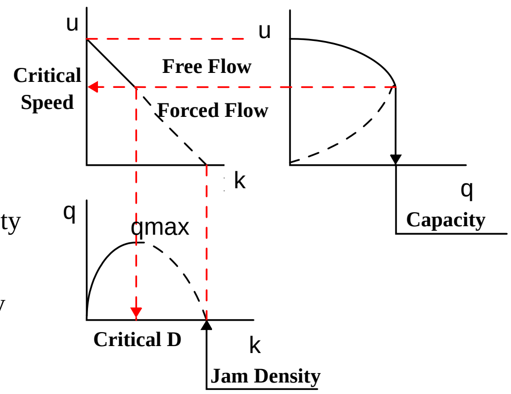
$$q = \left(\frac{k_j}{u_f}\right)(uu_f - u^2)$$

$$q = u_f k - \left(\frac{u_f}{k_j}\right)k^2$$



Greenshield Traffic Flow Model

- Unique Model Parameters:
  - Capacity (q_{max}): Max Flow
  - Critical Speed (u_c): Speed corresponding to max flow
  - Jam Density (k_j): Max Density
  - Critical Density (k_c): Density corresponding to max flow



GreenShield Traffic Flow Model

- Capacity q_{max} Estimation
  - At capacity, dq/du=0

$$q = \left(\frac{k_j}{u_f}\right)(uu_f - u^2)$$

$$\frac{dq}{du} = \left(\frac{k_j}{u_f}\right)(u_f - 2u_c) = 0$$

$$u_c = \frac{u_f}{2}$$

GreenShield Traffic Flow Model

- At Capacity q_{max}
  - Critical speed is, $$u_c = \frac{u_f}{2}$$
  - Substitute in 2, $$k_c = \frac{k_j}{2}$$
  - Substitute in 1, $$q_{max} = \frac{u_f k_j}{4}$$

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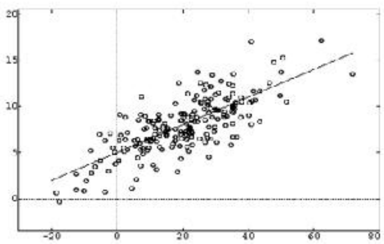
Calibration of GreenShield Traffic Flow Models

- Linear Regression Analysis

x	y
x_1	y_1
x_2	y_2
…	…



y=a+bx

y: dependent variable

x: Independent variable

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Calibration of Macroscopic Traffic Flow Models

- Linear Regression Analysis:
  - Minimizing the square of the differences between observed and expected values of a dependent variable.

y	x	x.y	x2
y_1 y_2 y_3 …	x_1 x_2 x_3 …	$x_1.y_1$ $x_2.y_2$ $x_3.y_3$ …	x_1^2 x_2^2 x_3^2 …
Σy $\bar{y} = \frac{\sum y}{n}$	Σx $\bar{x} = \frac{\sum x}{n}$	Σxy	Σx^2

$$b = \frac{\sum_{i=1}^{n} x_i y_i - \frac{1}{n}\left(\sum_{i=1}^{n} x_i\right)\left(\sum_{i=1}^{n} y_i\right)}{\sum_{i=1}^{n} x_i^2 - \frac{1}{n}\left(\sum_{i=1}^{n} x_i\right)^2}$$

$$a = \bar{y} - b\bar{x}$$

Calibration of Macroscopic Traffic Flow Models

- Linear Regression Analysis:
  - How to assess the model goodness of fitness?
    - ➔ Coefficient of Determination R^2

$$R^2 = \frac{\sum_{i=1}^{n} \left(Y_i - \bar{y}\right)^2}{\sum_{i=1}^{n} \left(y_i - \bar{y}\right)^2}$$

Where:

Y is the model predicted value of the independent variable

y is the actual value of the independent variable

Calibration of Macroscopic Traffic Flow Models

- Mapping to Greenshield's Model

y=a+bx

$$u = u_f - \left(\frac{u_f}{k_j}\right)k$$

u (y)	k (x)
u_1	k_1
u_2	k_2
…	…

a → u_f
b → $-u_f/k_j$

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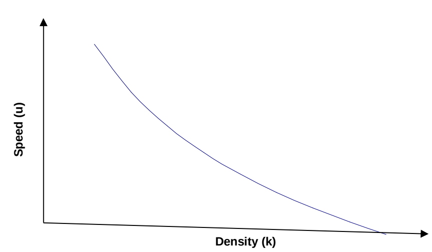
Greenberg Traffic Flow Model

- Fundamental Diagram
  - Analogy to flow of Fluids

$$u = c \ln \frac{k_j}{k}$$

$$q = ck \ln \frac{k_j}{k}$$

Where:
C is a constant



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Greenberg Traffic Flow Model

- Fundamental Relationship

$$u = c \ln \frac{k_j}{k}$$

Note that this relationship could be written as:

$$u = c \ln k_j - c \ln k$$

- u: dependent Variable (y)
- $c \ln k_j$: Constant 1
- c: Constant 2
- $\ln k$: independent Variable (x)

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